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QUESTIONING SKILLS OF PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS AND THEIR CONCEPTIONS OF PROBLEM AND PROBLEM SOLVING

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Abstract

The purpose of this qualitative study is to investigate the questioning skills and conceptions of four volunteer preservice middle school mathematics teachers within the context of problem solving and the relationships between their questioning skills and conceptions, if any. The data of the study was collected through clinical interviews. Firstly, each participant conducted a clinical interview with an eighth grade student within the context of a given mathematical problem. Secondly, participants were interviewed to reveal their conceptions on problem and problem solving process. The findings revealed that three of the four preservice teachers could not use appropriate questioning in the problem solving process. Additionally, none of them could make the formal definition of problem and problem solving phases. A consistent relationship between the conceptions of the participants and their practices in the clinical interviews they conducted with the eighth grade students was discovered.

Keywords: Conception, Functional Thinking, Preservice Mathematics Teachers, Problem Solving, Questioning Skills.

1. INTRODUCTION

Problem solving has taken its place as a subject studied with great importance for many years in the mathematics education literature. It is an indisputable fact that one of the important factors that play a role in the reflection of the researches, claims and the results obtained in the field of problem solving to school mathematics is the mathematic teachers. The knowledge and skills that the teachers should have are particularly emphasized in the literature. The studies showing that the pedagogical content knowledge of the teachers affects the learning processes of their students (Carpenter, Fennema, Peterson, & Carey, 1988: 385–401; Fennema & Franke, 1992: 147-164; Shulman, 1986: 4-14), clearly reveal that teachers should have both the content knowledge concerning both problem and problem solving (Silver, 1985: 247-266) and the pedagogical content knowledge as to how they can assist their students to gain problem solving skills.

Problem solving is one of the most important activities that we encounter in the learning environments set up in accordance with the constructivism principle adopted for more than thirty years in mathematics classrooms (O'Shea & Leavy, 2013: 293–318). It is observed in the literature that the roles of both the students (O'Shea & Leavy, 2013: 293–318) and the teachers (Martino & Maher, 1999: 53-78; O'Shea & Leavy, 2013: 293–318; Yackel, Cobb, & Wood, 1991: 390-408; Yackel, et al., 1990: 34-35) have been identified in the problem solving activities in these classroom environments. Teachers play a crucial role in the classroom environments where the students learn mathematics by discovering and reconstructing it themselves (Martino & Maher, 1999: 53-78). In rendering these environments, to be a watchful listener who can assess and predict the ideas that the student has formed in his/her mind and at the same time capture the opportunities that may help the student to take the ideas formed in his/her mind to the next step when necessary are deemed to be among the most important roles of the teacher (Martino & Maher, 1999: 53-78). Apart from this, teachers should assist their students to express what they think (Martino & Maher, 1999: 53-78; Yackel, et al., 1990: 34-35) and at the same time to encourage them to conceptualize situations in different ways (Yackel, et al., 1990: 34-35). It is deemed to be among the responsibilities of the teachers to focus on the ideas of the students, the responses they pose and the solutions they develop as well as determining what they can do regarding the different situations in problem solving and how the solution of the problem can be utilized in different situations is a necessity for teachers (O'Shea & Leavy, 2013: 293–318).

When the pedagogical knowledge and skills that the teachers guiding learning processes should have in classroom environments adopting particularly constructivism principle are considered, it is seen that one of the most important skills that teachers should have is their questioning skills (Burns, 1985: 14-17). Creating environments to encourage students in explaining their ideas, making comments or discussing each other's ideas in mathematics classes is closely related with the teachers' ability to pose questions

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(National Council of Teachers of Mathematics [NCTM], 1991: 24-34). Questioning skills of the effective teachers that are deemed to be one of the basic tools they have for many years (Way, 2008: 22-27) are also very important tools in providing students to construct knowledge and conceptual understanding (Purdum-Cassidy, Nesmith, Meyer, & Cooper, 2015: 79-99). The studies underline that the questioning strategies teachers use have a significant place in mathematics teaching (Mewborn & Huberty, 1999: 226–232). Furthermore, teachers' ability to listen to their students as much as they question their mathematical thinking plays a very important role (Martino & Maher, 1999: 53-78). The questions that must be asked and the points that must be paid attention in questioning are revealed in literature (Martino & Maher, 1999: 53-78, Purdum-Cassidy, et al., 2015: 79-99; Ralph, 1999a: 29–47; Ralph, 1999b: 286–296; Sullivan & Clarke, 1992: 42-60; Vogler, 2005: 98–103; Way, 2008: 22-27). For example, Ralph (1999a: 29–47; 1999b: 286-296) underlined the importance of asking clear and understandable questions to students, posing questions stimulating their thoughts and providing them with sufficient time for thinking and answering the questions. Purdum-Cassidy, et al. (2015: 79-99) underlined particularly the constructivist approach and drawn attention to the necessity of asking more clear questions, rendering student participation, requiring students to give longer answers and supporting high level thinking skills in such class environments. Way (2008, pp: 23), particularly taking into consideration the problem solving process, states that questions like "Could you recognized a pattern? Can you explain?", "How can this pattern be helpful to you in finding the answer?", "How would it be, if it were like...?" can be given as examples in assisting students to focus on a strategy in problem solving, to establish relations with their past experiences and to help to see the patterns and relationships. On the other hand, Way (2008: 22-27) named questions posed at the knowledge level as low level questions and underlined that these questions are deemed to be insufficient in stimulating mathematical thinking in problem solving process. Although the importance of questioning and the necessity to develop these skills have been underlined, the researches indicate that both preservice teachers (Henning & Lockhart, 2003: 46–57) and the teachers (Hiebert & Wearne 1993: 393–425; Klinzing, Klinzing-Eurich & Tisher, 1985: 63–75; Nicol, 1999: 45–66) experience difficulties in questioning. For example, Henning and Lockhart (2003: 46–57) state that numerous preservice teachers pose many questions one after another, that they do not pose follow-up questions, and they do not structure the questions based on the replies of the students. In the current study, the questioning skills and the conceptions of preservice middle school mathematics teachers on problem solving and teaching of problem solving, and at the same time the relationship between them are focused. When the mathematics education literature is reviewed, it is seen that the studies focusing on the relationships between the conceptions and practices have addressed teachers' conceptions on their teaching in the classroom environment (Thomson, 1984: 105-127; Thompson, 1992: 127-146). In these studies, it was seen that problem solving has been given a limited place. Studies focusing on conceptions particularly on problem solving and on the relationships between these conceptions and the teachers' questioning skills in problem solving processes could not be encountered in the mathematics education literature. For this reason, it is thought that this research may enrich specifically the problem solving literature by revealing the conceptions of preservice middle school mathematics teachers on problem solving and the relationships between these conceptions and their questioning skills.

1.2. Purpose of the Study

The purpose of this study is to investigate the questioning skills and conceptions of preservice middle school mathematics teachers within the context of problem solving and teaching of problem solving process and the relationships between their questioning skills and conceptions, if any. Answers to the following research questions within the context of this general aim are sought:

- How are the questioning skills of preservice middle school mathematics in the problem solving process?
- What are the conceptions of preservice middle school mathematics within the context of problem solving and teaching of problem solving?
- Is there a relationship between the questioning skills of preservice middle school mathematics and their conceptions within the context of problem solving and teaching of problem solving? If yes, what is the nature of the relationship?

The word of "conception" in the purpose of this study is used to address the general mental structure of a person covering his/her knowledge, beliefs, conception, preferences and views (Lloyd & Wilson, 1998: 248-274).

2. METHOD

Since an in-depth examination of the questioning skills of preservice middle school mathematics in problem solving process and their conceptions within the related context are aimed, this study has been designed as qualitatively.

2.1. Participants

The participants of the study were four volunteer students taking an elective course in the middle school mathematics teacher education program in a state university in Turkey. The participants, of which one is male and the others are females, were informed about their right to quit from the study whenever they wished and they were asked to sign a content form. Four preservice teachers participated in the study had already taken most of the courses on mathematics and mathematics teaching in the curriculum of teacher education program and they were about to complete their sixth term in the program. Their ages were between 21 and 22 and their CGPAs were between 1.95 and 2.87.

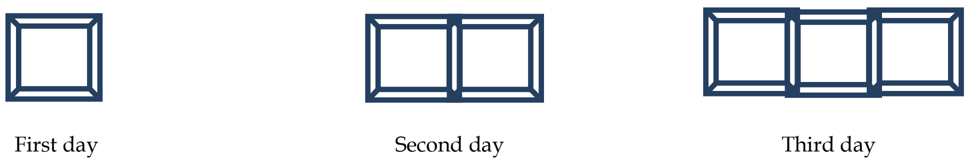
2.2. Data Collection

Data of the study was obtained through clinical interviews, one of the qualitative data collection techniques. Clinical interview is a technique frequently used in mathematical education that enables to reveal the cognitive structures of the students (Goldin 2000: 517–545). The clinical interviews used to obtain data in the study were gathered in two groups. The clinical interviews in the first group were the clinical interviews that the participants conducted with the eighth grade students. A short training was given to the participants on clinical interview technique. In addition, the participants were also informed that their objective in the clinical interviews they were to conduct was to reveal the mental activities of the students in the problem solving process and they were to use the questioning for this purpose. The participants were provided with the consent form for both eighth graders and their parents and the mathematics problem on which they make the questioning. They were also asked to record the clinical interviews with a voice recorder. The participants gave these voice recordings as well as the papers that students used for solving the problem during the interview to the researchers. The durations of the clinical interviews ranged between 15 and 27 minutes.

The second group of interviews conducted in the study to obtain data were the clinical interviews conducted by the researcher who did not teach the course with the participants. Following the interviews with the eighth grade students conducted by the participants, clinical interviews were conducted with the participants to reveal their conceptions on problem solving and teaching of problem solving and these interviews were recorded by the voice recorder. The duration of the clinical interviews ranged between 30 and 69 minutes.

2.2.1. The mathematical problem used in the interviews by the participants

The following problem was selected to be used by the participants in their interviews. In the literature, it is revealed that recursive thinking is necessary in pattern problems in the early grades, but functional thinking should replace recursive thinking in the progressing periods (Bezuska & Kenney, 2008: 81-97; Blanton & Kaput, 2004: 135–142; Moss, Beatty, Shillolo, & Barkin, 2008: 155–168). Since the interviewed students are at the eighth grade level, these students are expected to develop functional thinking that is necessary in the generalization of the pattern. For this reason, it is necessary for the participants to question the eighth grade students to lead them to functional thinking in making generalizations on the pattern.



A worker wants to build a fence as seen in the figure. The figure shows how much he can build at the end of each day. Therefore, how many pieces of wood the worker will going to use for building up the fence in a months' period?

2.3. Data Analysis

Following the completion of the clinical interviews, the transcriptions of the audio recordings were made by the researchers. The texts were analyzed by two researchers by using thematic analysis method separately (Gibbs, 2008: 38-56) and then common themes were determined. The analysis results of the researchers based on common themes were found to be consistent by 84% for the first group interviews and 91% for the second group (Miles & Huberman, 1994: 245-287).

3. FINDINGS

3.1. Questioning Skills of the Preservice Teachers

The research findings revealed that three (P1, P2, P3) of the four preservice teachers could not use questioning considering the problem solving phases and not enable the students to develop their thinking skills appropriate for the given problem. Although one of the participants (P3) among these three questioned the operations her student made; none of them could structure their questioning based on the responses of their students. Only one participant (P4) was able to use appropriate questioning helping the eighth grade student to construct a functional relationship in order to reach the aimed generalization in solving the problem by supporting the mental process of the student in the problem solving process.

P1 first asked her student to read the question and then she asked questions such as, "*What do we have to do now? Could you explain to me what the question asks? What does it ask from us?*" without allowing her student to think as well as without using a questioning regarding if the student understood the question or not. She led her student to develop a solution strategy immediately. In this process, P1, instead of leading her student, who could not understand the difference between the wood and the fence given in the problem, to discover the distinction them, she directly provided the information and expected the student to understand the problem. In spite of the fact that her student thought recursively, P1 guided her to check the algebraic expression that she obtained.

Student: Err, four fences are put together on the 1st day.

P1: Not four fences, four wooden pieces.

Student: … they already constitute a pattern. They increase three by three. … In order to find it, they already increase three by three. … wait a minute. Is it something like this? (writes down: (x+3)30). Is this correct?

P1: You should see that on your own, do you think it is correct? You may check it or something like that.

P1 reflected that she was not aware of the necessity of leading her student who was thinking recursively to functional thinking. Instead of questioning her student to make her understand the functional relationship in the pattern, P1 suggested the student to find the solution by trying. By this guidance, the student tended to reach the solution by trying and began to make meaningless manipulations. In this process, it was seen that P1 questioned the independent variable in the algebraic expression written by her student. Although the student implied that x was equal to the constant in the first step of the pattern, P1 again did not lead her student to discover her faulty thought. P1 who proved that she was not aware of her student's thought conflicting with the variable concept during the interview also reflected that she was not aware of functional relationship in this problem.

Student: I will find it one by one, then I do multiplication or division and find a solution.

P1: What is x here?

Student: You see, it is the unknown for example. You see there are 4 pieces of fence; we write x instead of it plus 3. It goes like that.

P1: You think that it goes like that all the time? It continues like x+3 each time? Try it for the 2nd day, is it okay?

P1's student reached the solution by drawing the squares one by one as seen in Figure 1 and by counting the sides. Then P1 asked her to reach the solution from another way by saying "*You could try it from another way by not using a variable…*". Therefore, it was seen that P1 was not aware of that she should have guided the eighth grade student to algebraic thinking.

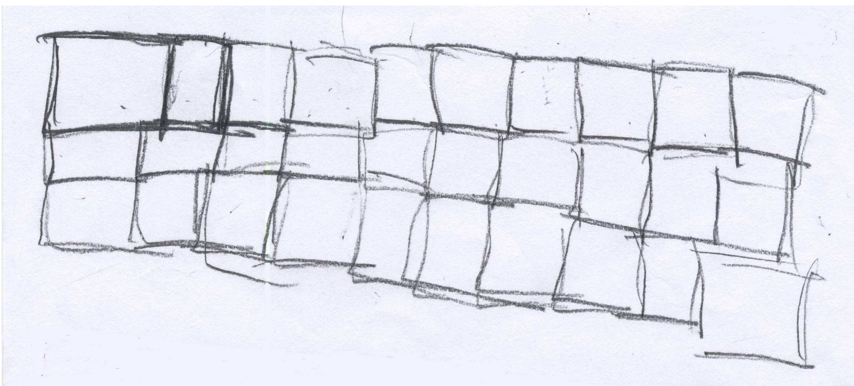


Figure 1: The solution of P1's student

P2 was another preservice teacher who first led his student to read the question and then think about it as much as she wished. When P2 asked the student what she had understood, it was seen that the student could not fully perceive the distinction between the wood pieces and the fence in the problem. In this process, P2 guided his student by using some expressions such as "*try to make sense of it again,*" "*how could you address the problem by using the givens and the asked?*", and "*how would you express it by writing it down?*" When this guidance did not work out, P2 gave information to his student and made the distinction between wood pieces and the fence explicitly. Then it was seen that the student thought that the total number of wood pieces dependent on the number of fences. P2, instead of leading the student to understand the relationship between the number of steps and the number of fences, and thus to functional thinking, he led the student to read the problem once again to find the solution.

Student: (Thinking…). If he uses four pieces of wood for one piece of fence…

P2: Let's express these at the same time.

Student: (Writing…). If he uses four pieces of wood for one piece of fence, then he will use eight pieces of wood for two pieces of fence. He will use 12 pieces of wood for three pieces of fence.

The student who was not subjected to questioning supporting her mental process, discovered the relationship between the number of fences and the days through thinking out loud and started to take the independent variable as the number of days.

Student: As the number of days increase…the pieces of wood used are four times the number of days ……..

Student: As the days pass, the number of fences equals to the number of days.

Following this stage, P2's student was able to discover the functional relationship in the pattern, but P2 asked some questions such as "*Do you think it is the correct answer? What we have tried to do? What type of modeling did we use? Could you tell all about this?*" instead of leading his student to find the algebraic expression of the relationship. Consequently, the student could not make any sense of these questions and left them unanswered.

Although P3 could not use appropriate questioning, she reflected that she was aware of the necessity that her student had to make algebraic generalizations. At the phase of understanding the problem, P3 made various guidance such as, reading out the problem, reading it silently, expressing what she understood from the problem in her own words, questioning what was being given and what was being asked in the problem. Then, she posed the question of "*are the givens sufficient for you to solve question?*", however, it was seen that she did not question the reason for the student's answer which would have enabled herself to think the relationships in the problem. Then, she led her student to solve the problem.

P3: Are the givens sufficient for you to solve the problem?

Student: Yes, they are, I can solve it.

P3: Then, let's start solving it.

P3 noticed that the student mixed the pieces of wood with the fence, and gave information to the student more than required by using statements such as "*… I think what you talk about are not pieces of wood. They are fence…*," "*… each day they have used 4 pieces?*", and "*Count the second day. Then eight pieces of wood have been used*." The student noticed her mistake as a result of the guidance of P3, but this time the student began to think recursively by the guidance of P3. In this process, P3 reflected that she was aware of the necessity that her student, who tried to reach the solution arithmetically through recursive thinking, should have been led to algebraic solution; however, she could not lead her to algebraic thinking. It was seen that P3 tried to lead her student to algebraic solution by asking her to think "*another solution way*".

P3: What will happen on the fourth day for example?

Student: 13 pieces. Shall I write them down one by one?

P3: Go on as you wish.

Student: (Starts writing…)

P3: Are you going to write until the 30th day?

Student: Indeed, there is a simple solution.

P3: Can you think of that?

Following similar utterances, the student again tended to the arithmetic solution and reached the solution by arithmetic means; and explained the solution verbally as "*since it increases three by three in a month's period, I multiply 30 by 3 and it makes 90; and since there is one piece more from the 1st day, I will add 1 to 90*." Following this explanation of the student, P3 said that "*you have just convinced me about the solution by writing them down one by one, but it would be better if you could solve it in the practical way and showed this way as the final resort as its validation.*" And by saying this, she characterized the algebraic solution expected from the student as the "*practical way*".

P4, who was the only participant using appropriate questioning in the problem solving process, asked questions aiming to encourage her student to construct the functional relationship given in the problem. Moreover, she structured the questions based on the responses of her student. She also reflected that she was aware of the necessity to solve the problem in the algebraic manner. P4 first read the question herself and then had her student to read the question. After her student read the problem P4 asked her whether she understood the question and also ask her to tell the problem. Following this, P4 led her student, who was in the thinking process on her own, to draw the figure to her paper in order to have herself to see the relationships in the problem. At this stage, the student tended to find the solution by using proportional reasoning and P4 questioned the thinking way of the student with the "*how?*" question and allowed her to explain her thought.

Student: I first came up with the idea that I will go on with the pattern.
P4: How will you do that?
Student: If there are four pieces on the 1st day, how many will be done in 30 days? If we use ratio and proportion I thought of 120. If four pieces are made in one day, then 120 pieces will be done in 30 days.

After this stage, P4 questioned her student by saying "*okay then, if there are four pieces in one day, then should it be eight pieces in two days' period?*" and had the student notice that the result student obtained is wrong. In this case, P4 reflected that she took functional thinking as a goal by leading her student who was thinking in a recursive manner to the independent variable of "*day*."

Student: I mean, this has a rule. I think of it as a pattern. This has a rule, and I should find that rule.
P4: Okay, how can you find it?
Student: First of all, I will find the number of fences used each day. … There is a difference of four between these (she is pointing out the 1st and the 2nd days). There is a difference of two (pointing out the 2nd and the 3rd days). ….. When it is two times, it is not like that. There is an increase in the increases, but… it decreases… hmmm.
P4: Okay, can you construct the pattern in another way?
Student: In another way, what do you mean?
P4: I mean, would it be necessary to use the days also?

It was seen that the student started to take into consideration the number of days by leading of P4, and she focused on finding the relationship between the number of days and number of fences. At this process, P4 questioned every step of the student, asked her to explain how she made it and posed questions encouraging the student to think. P4 allowed her student to try every possibility that she could have tried while she was trying to find out the pattern rule by focusing on the differences and times between the number of days and fences. When the student understood that she could not solve the problem by focusing on the differences, P4 tried to make her to be aware that she should have use a variable by asking the student "*what is the variable here?*" In this process, it was seen that, P4 was able to enable the student to determine the dependent and independent variables through her purposive questioning and led her student to the desired point.

P4: If you cannot proceed through the differences, how can you do that?
Student: I can proceed again with four, eight, ten; I mean, I can proceed with even numbers.
P4: So, what is the variable here?
Student: Variable is the number of fences.
P4: How does it change? It changes based on what?
Student: Based on the number of days. If I can find an "n" for that. We were taught like that.
P4: Why you can call it an "n"?
Student: I can call the number of fences as "n". Or I can call the amount of increase as "n". We cannot call the number of days as "n".
P4: Why you cannot call them as "n"?
Student: Since it is something already determined… I mean, we cannot call the number of days as "n". It increases like one, two, three, and four. The variable here is the number of fences. This is why we should call the number of fences as "n".
P4: Now, what happens when we say "n"?

After P4 led her student to functional thinking by her questioning, her student was able to reach a generalization. Finally, P4 completed the interview by using appropriate questioning taking into consideration the evaluation phase of the problem solving process.

3.2. The Conceptions of the Preservice Teachers and The Relationships Between Their Conceptions and Questioning Skills

The results of clinical interviews regarding the conceptions of the participants indicated that none of the four preservice teachers could make the formal definition of problem using mathematics education terminology. However, it was seen that only one participant (P4) had the theoretical knowledge with respect to what makes a situation a "*problem*". The other three participants (P1, P2, P3) stated that a problem is a situation faced in daily life tried to be solved by the individual and also they tried to explain the problem solving phases as a problem solver based on their own experiences. The other preservice teacher (P4), on the other hand, displayed that she was aware of the problem solving phases and tried to explain them from the instructor viewpoint, yet she could not entitle the phases in accordance with the terminology. In addition, all of the participants mentioned the role of problem solving in mathematics education both as a goal and a tool in a different manner, P4 reflected that she had more pedagogical knowledge than the other participants.

When the relationships of the questioning skills and the conceptions of the participants were examined, a consistent relationship between the conceptions of all of the participants and the practices in the clinical interviews conducted with the eighth grade students was discovered. It was seen that the conceptions of the preservice teachers regarding the problem and problem solving process reflected on the questions they posed during the interview they conducted and on their guidance.

3.2.1. The case of P1

P1 perceives the problem situations as situations faced in the daily life, which needs to be solved and constitutes a problematic situation for her. In parallel to this, the problem examples that she gave in the interview were from daily life. By the examples P1 gave, she stressed that the condition for a situation to be a problem is that it necessitates a solution. When she referred to mathematical problems, she always thought of the four operations and this proved that her mathematical problem image involved the word problems.

P1: When we think about what we can buy with the money we have is a problem situation. Solving this issue is problem solving. We produce a solution for the problem. In other words, we make additions and subtractions and we think of whether to buy this or that...

…….

P1: I have an examination tomorrow. How much time do I have to study before the examination? We make a mathematical operation for this problem. ….How many hours do we have? … We produce a problem and we bring about a solution for that.

When P1 was questioned about the phases of the problem solving process, she formerly stated that this process was composed of eight stages, but then her latter explanations were inconsistent with the former statement. When her knowledge with respect to problem solving process and its phases was questioned, she reflected her experiences as a problem solver. This participant stated that she would directly go to the solution in the process and she would use four operations to reach the solution. Consequently, she proved that she did not have any formal knowledge regarding the phases of problem solving process.

Interviewer: What has to be done first in the problem solving process?

P1: One has to see what is being asked, and then… one has to find a solution

Interviewer: Yes.

P1: Maybe one has to think about what kind of a solution it would be.

Interviewer: Then?

P1: And then… one thinks about which method will be used to solve the problem, and solves it.

At this point, P1 said that she would head for directly to the solution of the problem and she underlined that she had been taught like that. As it was explained in detail in the previous section, P1 guided her student rapidly to solution without allowing the student to think about the problem in the interview that she conducted with the eighth grade student.

P1 stated that problem solving is very important and underlined her idea with the statement of "*mathematics is actually a problem*." Besides, she stated that problem solving is important in terms of understanding the mathematical subjects, however, she proved that she thought of it not as a teaching method, but rather as a support to the learning of the mathematical subject by solving problem related to the subject.

Interviewer: Okay then, what does problem solving contribute to students?

P1: We can have them solve problems to understand the subject.

When this participant was questioned on her role as a teacher in the problem solving process, she stated that she would lead the students to the solution following activities such as reading the problem, identifying the givens and the asked in order make them to understand the problem. When the interviewer asked about what could be done for the students, who did not understand the problem, it was seen that this participant had the opinion that she would explain the problem by summarizing it instead of using questioning for the students to understand the problem. What is striking here was the fact that this participant provided her student with extra information instead of using questioning in order to make her to think in her own interview. Although the participant stated that her role as a teacher to be a "*guide*" in the problem solving process, when she was questioned on what kind of a guide she would be, she stated that she would mostly ask questions that would start with "*what*" instead of posing questions to guide them to the essential ways of thinking.

Interviewer: What kind of a guide you would be?

P1: Indeed, I would ask about their thoughts. What do you think? What would you like to do? Where would you like to start? What do you have in your mind? I would ask these and the like…

In here it was seen that the questioning style of the participant as a "*guiding role of the teacher*" with her own words was consistent through questionings such as *"… What do we have to do now? ... Could you express what is being asked in question? What does it ask from us?"* in the interview that she conducted with the eighth grade student. In addition, it was seen that this participant was under the influence of the mathematics education experience of the past years and she reflected this on her own teaching approach. P1 did not reflect a negative attitude to her own experiences in the elementary and secondary school years, but just presented the situation in those years by such statements as "*they taught us like that in our school years*."

Interviewer: What is done first when a mathematics problem is encountered?

P1: First… Indeed, I would first resort to the solution.

Interviewer: You resort to the solution?

P1: Because we were taught like that when we were students at elementary school… I mean, it was more important to solve the problem rather than thinking about it.

Interviewer: But, what do you need to do to solve the problem?

P1: I mean, I would go directly through the operations.

Interviewer: Directly through the operations?

P1: Exactly, I would do the operations, addition, subtraction…

3.2.2. The case of P2

This participant could not provide any terminological information regarding what the problem is and described it as a "*difficulty that an individual has encountered and has tried to solve*". When he was questioned as to when a situation was seen as a problem, he made judgment such as "*there should have been some unknown things. I suppose the instructor told something like that…*" Additionally, P2 did not have any theoretical knowledge with regard to problem solving phases, and interpreted the problem solving process as a problem solver like P1. He tried to explain the problem solving process as the "*hypothesizing and testing the hypothesis*", which is among the scientific process skills.

P2: When I faced with a mathematic problem, first, I should read it; I mean I should try to make sense of it after reading it. After I have made a correct sense of it, I should write down the givens. Then, in the light of the givens, what is the asked? I should also write down the asked. I mean, what kind of a relationship should I establish? I should try to find something like hypotheses. Of course, I should do this on the basis of my mathematical knowledge. Then, I should start to solve the problem, I mean; I should try to test if the hypothesis in my mind is correct.

When the importance and the place of problem solving were questioned, P2 linked the importance of problem to the importance of four operations in our daily life and also he stressed that problem solving supports the understanding of the mathematical subject. Based on these conceptions it was thought that, P2's problem image was composed of word problems. His views on that "*the weight of problem solving in the mathematics education programs may be decreased*" were found quite remarkable.

P2: It is not possible not to teach, but the weight of problem solving may be decreased. For example, if problem solving constitutes 80% of the program, this can be reduced to 50%.

P2 expressed that his role as a teacher is to make the students to think in the problem solving process and stated that he would question the students dependent on the "*what?*" question. Even though he argued that he should make the students to think, he reflected that he did not have the knowledge of questioning in the problem solving process.

P2: I ask questions to the students related to the verb and I try to make the student find the solution. For example, it says "a fence is built" in the problem. I immediately ask "what is being built?" "what is asked?"…

This participant did not reflect that he had knowledge of questioning and also he reflected that he was not aware of the necessity to guide the student to functional thinking during the interview. This participant resorted to giving extra information to his student who had difficulties in understanding the quantities given in the problem. A striking point was that he thought that as a teacher he should not interfere with the student while the student was writing or solving the problem. It was also seen that, P2 left his student alone in the process by using such expressions as "*yes*", "*okay, let's continue*" and the like and did not pose any questions to her.

3.2.3. The case of P3

When questioned on what a problem was, it was seen that P3 first perceived the meaning of problem in our daily life and tried to define the problem through an expression: "*Anything that we encounter, we can not solve and ask for an answer in life is a problem*". When her conceptions on mathematical problem were questioned, P3 tried to define a mathematical problem as "*a question which includes givens and the asked … I mean, how I can explain it, I don't know*". In order to probe the conceptions of P3, a question "*Well, is every question a problem*?" was asked to P3 and she replied as "*every question is not a problem. For example, in the x+2=4 equation, the question of "what is x?" is not a problem, because it does not necessitate thinking. It asks from us to find x directly. The value of x is apparent. But, a problem asks from us to equate that by thinking a little bit*". P3 stated the condition to be a problem is to make a person to think.

P3 could not make the definition of the problem in accordance with the mathematic education terminology and also she was lack of formal knowledge regarding the problem solving phases. Although P3 listed the phases of problem solving as the steps that a student has to follow, she presented these phases as a problem solver by putting herself into the shoes of the student and based on her own experiences.

P3: One will read the question first, then s/he will have the data, and then s/he will find out what is asked from us; what is asked is important. Then, after the student has revealed these, s/he will think about how I can use these data. The student will formulate the solution on the basis of what is asked by considering how I can use the data.

This participant expressed the things that the student has to do regarding the problem solving phases by statements such as: "*S/he has to understand, absolutely; s/he has to think about the data in his/her mind, I mean, s/he has to think how can I use the data there, how can I link them, how can I establish a relationship with the given data, how can I reach the solution…*" The reflections of these conceptions of P3 were observed in her own clinical interview. P3 also reflected that she was aware of the necessity of making algebraic generalizations by the student in the problem solving process, and asked her student to read out to question and read it silently and guided her to express what she understood from the problem in her own words. Additionally, this participant asked questions such as "*are the given data enough for you to solve the question*?" in parallel with the idea of guiding the student to interrelate the given data in the problem which she clearly expressed in the interview.

This participant expressed her conceptions related to the place of problem solving in mathematics education with the following statements and clearly presented the importance of problem solving: "*I think its place is huge, I mean significant. Even it is so important that it cannot be explained… I think mathematics should entirely be composed of problems*". P3 clearly expressed that problem solving is very important and it was seen that she pointed out to the use of it as a support in the development of problem solving skills and the practice of the learned subjects as can be seen in the following quotation;

P3: Problem solving should ensure thinking. S/he will think in a way to develop his/her mind. Whether s/he uses the previously learned subjects in the problem solving process. Problem solving improves this. S/he will be repeating what s/he has learned.

When the conceptions of P3 regarding her role as a teacher in problem solving process were questioned, she firstly expressed that she would teach how to use the givens within the context of problem, that she would not give the answer directly and she would examine the ideas of her students. When this participant was questioned particularly on the conception as to how she would examine ideas of her students, it was seen that, this participant adopted the idea of asking questions to her students. She indicated that she would ask questions related to the given and the asked in the problem as well as questions starting with "*how*" and "*why*." This participant said that she would ensure that her student would reach to the solution by her questions however, when the student cannot succeed, she clearly said that she would help the students. Another remarkable point here was that, although this participant underlined that the problem should make the student think, her statements as to "*how s/he will reach the answer?*", "*I will ensure that s/he would think about the answer*" led to the thought that this participant regarded the problem solving process more as a solution-oriented manner rather than a process-oriented one.

P3: The student first should know what is asked from us; what is given and what is the asked? … I will have him/her write down these data on the board. What is the relationship between these data? How s/he will reach the answer with these data? I will make him/her to think about the solution. I will definitely ask questions: How are we going to use this? For example, why do we use this like that? I will definitely ask questions to make him/her to reach the solution. If the student still cannot solve the problem, then I will help him/her.

It was observed that, the role that P3 expressed as a teacher was reflected in her guidance in her own clinical interview in three different dimensions. First of all, P3 stating, "*if s/he cannot solve problem, then I will help him/her,*" guided her student by providing information more than required instead of using appropriate questioning to make her to discover the quantities in the problem on her own. Secondly, it was seen that, P3 posed questions to her student such as "*What is given in this problem?*" "*What is asked?*" "*How did you find it?*" "*Why did you multiply it with 4?*" similar to those which she exemplified types of questions she would ask. Thirdly, although P3 reflected that she was aware of the fact that she should have guide her student to algebraic solution, the acceptance of student's arithmetic solution by saying that "*you have convinced me on the solution by writing them down one by one*" made us think that the reason behind this acceptance is the fact that P3 regarded problem solving process more like a solution-oriented manner.

3.2.4. The case of P4

When the conceptions of P4 were questioned regarding what a problem is, it was seen that that she immediately perceived it as a mathematical problem differently from the other participants. Although P4 could not make the problem definition as used in the mathematics education terminology, she defined it as "*the questions that we ask the students to find the solution or search for the various solution ways.*" In this context, when these conceptions of P4 were questioned more deeply it was seen that she had the knowledge about the important characteristics of problems used in school mathematics. P4 also reflected that she had knowledge that being a situation to be a problem varies from one person to another by expressing that "*each grade's problems should be different from the other*."

P4: … the difficulty level is important. … a problem for a sixth grade student is not a problem for an eighth grade student. … a problem should make the students think and challenge their minds.

Although P4 could not reflect the information on mathematics education terminology related with problem solving as expected from a teacher candidate, she caught the attention as a participant who was able to use expressions such as strategy, problem solving method and making concrete. In addition, when P4 was questioned regarding the phases of problem solving, she responded not as a problem solver as other participants, but rather from the viewpoint of a person who teaches problem solving. P4 reflected her terminological knowledge related to problem solving process with statements such as "*first, we explain the problem*", "*the solution is reached in accordance with the phases*."

P4: First, we tell the problem of course, and then we think about how to solve the problem…

………….

P4: Then, we reach the solution in accordance with the phases, I mean, are there any different solution ways?... Will we be able to reach a certain solution?

………

P4: While we are trying to solve, we think about the ways we are able to solve the problem. We decide on the solution way and then we solve it…

………

P4: After we solve it, we can check it again…

When P4 was questioned about the place of problem solving in mathematics education, this participant emphasized that problem solving should force one to think, and she underlined particularly the purpose of problem solving in developing skills. Additionally, she reflected that she had the knowledge that problem solving can also be used to teach the subjects.

P4: … problem solving enables us to think more easily, and also think about different ways and develop strategies.

….

P4: It enables students to see different ways, if s/he solves more problems, it can also enable the students to practice more.

….

P4: If s/he can solve the problem, I think it means that s/he has understood the subject.

P4 underlined the role of problem solving as a goal within the context of gaining problem solving skills and she argued that the phases of problem solving should not be disconnected from each other and problem solving should be done in accordance with the answers the students have given. In addition, P4 placed problem solving to a different place than the learning of other mathematical concepts and reflected that she was aware of the development of different ways of thinking and solution methods in this process.

P4: Since there are various types of problems, it is necessary to develop different ways for each problem… Like the other subjects, there is not only one method …. there is a different solution method for every problem. The student can resort to different thinking ways…

When this participant was questioned regarding to her role as a teacher in problem solving process, she presented totally a different picture than the other participants. The comprehensive and detailed conceptions of P4 proved that she was aware of the pedagogical responsibilities as well as she was equipped as to how to fulfill these responsibilities in a purposive manner. P4 reflected that she viewed the problem solving process from a teacher's perspective throughout the interview and expressed that she had to make the students understand the problem, and as a prerequisite to this she said that "*but first, I have to understand it.*" By saying this she proved that she was aware of the fact that teachers should have content knowledge in mathematics. In addition to these, P4 also underlined that students should not start to solve the problem before they understand it.

P4: I should make them understand the problem first, but before that I should understand it myself…

….

P4: First, I should study myself… When I understand that they also have understood, I should start to solve…

P4 emphasized the importance of the phase of understanding the problem in the problem solving process and consistently with her conception she had her eighth grade student first to explain the problem in her clinical interview. Afterwards, it was seen that the student started to think on her own and P4 guided the student and told her that "*if you wish first draw the figure*" in order to ensure that the student understands the relations in a better way.

P4, who was aware of the fact that she should question the thinking ways of the students, also reflected that she was aware that she has to structure the questions based on the responses of the students by saying that "*if I ask questions to the students, I should question them in line with the answers they give.*" The reflection of this conception that P4 adopted was observed clearly in her own clinical interview. As it was detailed in the previous section, this participant structured almost all of the questions based on the responses that the student gave. P4, who implied the phase of evaluation, stated that she would first solve problems similar to the one they were solving at this phase, and then she would solve different problem examples with the students by changing the variables. It was seen that P4 adopted the conception of *problem solving improves thinking*. Being aware of this responsibility as a teacher, she reflected the purpose of changing the variables by the statement of "*if the variables change can the student think differently?*"

P4, who was questioned on what she would do as a teacher when the solution was reached, stated that she would ask her students to repeat the problem solving process and by this way she would be convinced if the students have understood the process. She also proved that she was aware of the fact that she has to ensure that the students interrelate the solution with the data within the context of the problem by asking questions such as "*what we have tried to find out*?" at this phase. Similarly, this approach of P4 was observed in her own clinical interview. In the interview, she led her student who reached the algebraic solution to review the problem solving process and to requestion her ways of thinking by posing questions such as "*now, what have you done, to repeat again*?", "*how would it be if you write it down correctly?*", "*would you be able to solve it without using an "n" while solving in this way in finding the 30th day?*"

4. DISCUSSION AND CONCLUSION

The results related to the questioning skills of preservice middle school mathematics teachers in the current research support the findings regarding the difficulties preservice teachers in using questioning in the literature (Henning & Lockhart, 2003: 46–57; Hiebert & Wearne 1993: 393–425; Klinzing, et al., 1985: 63–75; Nicol, 1999: 45–66). When the questioning skills are examined in the problem solving process, it has a particular importance in mathematics teaching. Problem solving has a crucial place in the mathematics education as a tool in terms of teaching methods and as an aim from the point of the development of problem solving skills. Therefore, it is necessary for the mathematics teachers to use problem solving as a tool in the learning environment and to aim to develop problem solving skills of the students (Martino & Maher, 1999: 53-78; Sullivan & Clarke, 1992: 42-60; Way, 2008: 22-27). Ultimately, the pedagogical education that preservice mathematics teachers take within the context of problem solving should bear the characteristics to furnish them with these questioning skills (Moyer & Milewicz, 2002: 293–315). In this study, it was seen that, only one of the four preservice middle school mathematics teachers (P4) could use the appropriate questioning that would enable her student to reach the intended generalization in the problem by supporting her mental process in the problem solving process. In addition, it was remarkable that the only eighth grade student who could reach the generalization in the problem was the student this participant interviewed. On the other hand, although the student, P2 interviewed, had the thinking way to reach the generalization in the problem compared to the other students, she could not generalize the relationship in the problem. It was thought that the reason was that the student could not make sense of the guidance of P2 who did not support the thinking process of her by providing information more than required and using statements such as "*make sense of it again*", "*what type of a modeling have we used*?" The findings of the study have revealed that preservice teachers' failure to use appropriate questioning results in failure in managing and accomplishing the problem solving process in a successful manner as underlined in the literature (Henning & Lockhart, 2003: 46–57; Martino & Maher, 1999: 53-78; Purdum-Cassidy, et al., 2015: 79-99; Ralph, 1999a: 29–47; Ralph, 1999b: 286–296; Sullivan & Clarke, 1992: 42-60; Vogler, 2005: 98–103; Way, 2008: 22-27).

In the literature, it has been emphasized that the indicator of success in problem solving within the context of developing problem solving skill is not to solve the problem correctly, but rather to emphasize the thinking ways in the problem solving process (Harel, 2001: 1-12; Harel, 2007: 263-280). In this study, the guidance of the students by most of the participants was to find the solution of the problem in a rapid manner supports the results obtained by Harel and Lim (2004: 25-32). Harel and Lim have reached to the conclusion that preservice mathematics teachers focus on the reaching the solution of the problem and neglect developing their students' problem solving skills.

The results of the research concerning the conceptions of the preservice teachers within the context of problem solving and the teaching of problem solving were consistent with the questioning skills of these participants in the problem solving process. The existence of a consistent relationship between the questioning skills of the preservice teachers and their conceptions is also consistent with the results of the studies in the literature that put forward the conceptions of particularly mathematics teachers affect and shape the practices in the learning environment in a strong manner (Thompson, 1884: 105-127; Thompson, 1992: 127-146). The preservice teachers who could not use appropriate questioning in the problem solving process could not reflect any terminological knowledge. The participant, who was able to use a successful questioning in the problem solving process, proved to have the terminological knowledge in the mentioned context with her conceptions. It was seen that, the participants, who could not use appropriate questioning, tried to interpret the problem solving process as a problem solver, while the participant, who was successful in questioning, explained the problem solving process from the viewpoint of a teacher and more in terminological terms than the other participants. On the other hand, none of the participants, including P4, could define the mathematical problem precisely, explain the conditions of a situation to constitute a problem and the phases of problem solving process in full. Therefore, there is a lack in the mathematics teacher education system within this context. As a result, it becomes evident that it is necessary to review the mathematics teacher education programs in terms of pedagogical knowledge within the context of problem solving and improving their questioning skills. In line with the findings of this study, a research study aiming to monitor the development of the knowledge and skills of the middle school mathematics teacher candidates in this context in a teaching experiment intended to introduce the preservice middle school mathematics teachers by integrating the terminological knowledge within the context of problem solving with questioning methods may be suggested.

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